

BIOPHYSICS OF COMPLEX SYSTEMS MATHEMATICAL MODELS

CONTROL AND ADAPTATION (Evolutionary aspect of the problem of control)

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A hypothesis is discussed on the connexion between control and adaptation. Control is the form of adaptation at the level of the "collectives" essentially related to breakdown into "passive-stable" and "adaptive" populations and appearing in special critical conditions. A mathematical model of the evolutionary origin of control is proposed. The difficulties of studying this model and the importance of the boundary cases allowing investigation by special methods are indicated.

INTRODUCTION

THE concept of control has been widely discussed in the scientific and, in particular the popular scientific literature. It is even difficult to list the various nuances, some even contradictory, with which different authors surround this concept. Three main

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very extensive sets of studies exist within which the term "control" is understood quite uniformly.

In the theory of optimum regulation the discipline, in essence purely mathematical, there is a distinct definition of the concept of "control". Thus, the free parameters are called $u(t)$ in the set of equations determining the dependent variables $x(t)$:

$$dx/dt = a(x, u).$$

The main emphasis of this definition is the independence of the controlling parameters $u(t)$ of the controllable variables $x(t)$.

Other aspects, for example, the effectiveness of control, i.e. the value of the "force" the necessary to obtain the required result Δx are of secondary importance. The effectiveness may in particular be as small as desired and this does not change formally the irreproachable division of the variables into controlling and controllable.

Control in biology, and in particular, biochemistry is interpreted differently and in a somewhat less definite manner. Instead of the term "control" often one uses the term "regulation" meaning the process or the result (or both) of the activity of enzyme systems of the most varied type. The emphasis of such an interpretation of control is different. Coming to the fore here is the effectiveness of low concentrations of substance-regulators and their specificity. It becomes impossible to insist on the strict independence of the controlling agents and the controllable—both are concentrations of different chemical compounds involved in the general reaction.

The concept of control has the most abstract meaning in the third group of studies, cybernetics, where it is equivalent to the concepts "communication" or "signal". This means a dual idealization. The idealization consists firstly in the assumption that the signal causes the required change without any expenditure of energy or materials. On the other hand, the total independence of the control from the controllable systems is postulated. The latter abstraction is somewhat softened by introduction of the concept of "feedback" when the action of the results of control on the subsequent signals is assumed. Such an approach introduces (because of the superfluous rigidity of a discrete scheme, so popular in cybernetics) a forced delay (by one "stroke") quite unnecessary not only in actual systems but even in other mathematical models.

The differences enumerated are quite serious. They make doubtful the existence of a single concept of control suitable for all applications. As with any scientific concept the concept of control describes extreme boundary situations. In natural science such situations are realized only approximately. Therefore, the fundamental methodological problem of the boundaries of applicability of the concept of "control" arises.

The problem concerns not the formal strictness of terminology. The problem of the boundaries of applicability has a quite fundamental evolutionary form. Since the properties of the systems which we now call "control" or "regulation" appeared not immediately but traversed the long path of evolutionary development then control must have its own evolutionary precursor.

The article puts forward and argues for a hypothesis that control is the development and extreme asymptotic form of adaptation.

1. MATHEMATICAL MODEL

In a previous paper [1] it was shown that the concepts associated with the phenomenon of adaptation such as "work capacity", "shock", "rest", "adjustment", etc, may be (in the simplest form) modelled by a system of only two equations. However, these equations must be essentially non-linear and contain parameters necessarily including one small parameter ε .

$$\left. \begin{aligned} dy/dt &= b(y, z, \varepsilon, \alpha) \\ \varepsilon \cdot dz/dt &= c(y, z, \varepsilon, \alpha) \end{aligned} \right\} \quad (1.1)$$

But as against this there are ordinary differential equations for which mathematicians have developed a fundamental qualitative and, in part, quantitative theory.

This circumstance important for the rest of the paper may be understood by reference to another paper by the author [2]. In it, it is shown that the behaviour of even a very complex system is sharply simplified in a critical situation. In conditions when the system is threatened by loss of stability (in any sense whatsoever) the number of determining variables dwindles to two (when the system passes through a complex root, i.e. through an oscillatory regime with loss of stability) and sometimes even to one. Therefore, the main features of the behaviour of all systems are determined not so much by their internal structure but by the character of the critical conditions into which these systems may fall. The hypothesis governing this paper is that control is one of the most effective forms of adaptation at the level of "collectives". Such biological collectives may be tissue cultures or colonies of unicellular organisms, herds of animals flocks of birds and the "state" of collectivized insects.

The main step in the construction of the model is the assumption that the behaviour of such collectives may be modelled by a set of a large number of uniform equations:

$$\left. \begin{aligned} dy_i/dt &= b_i(y_i, z_i, \varepsilon, \alpha) + \delta B_i(y_1 \dots y_n, z_1 \dots z_n, \alpha) \\ \varepsilon \cdot dz_i/dt &= c_i(y_i, z_i, \varepsilon, \alpha) + \delta C_i(y_1 \dots y_n, z_1 \dots z_n, \alpha) \end{aligned} \right\} \quad (1.2)$$

Subscript i , is the number of the specimen, b_i and c_i describe the behaviour of the individual taken separately while B_i and C_i give the interaction of individuals with each other and the external environment represented by the parameter α .

In this system a further small parameter δ appears. Unlike ε , characterizing the internal properties of the individuals, this parameter gives the value of their interaction with each other and is in a certain sense a measure of the individuality of each individual. The corresponding large parameter

$$T = 1/\delta, \quad (1.3)$$

gives by order of magnitude "the lifetime" of the separate individual (measured by the value of the characteristic cycle of vital activity).

The model obtained is characterized by three time scales. The mean scale, time of the order of unity corresponds to the inherent times of the vital activity of the individual. We shall nominally call the corresponding changes "physiological". The small times $\Delta t \sim \epsilon$ are the times of adaptive jumps in the sense in which this phenomenon is considered in [1]. Finally, there is a large time given by formula (1.3) during which significant changes occur for each separate individual.

2. EVOLUTION OF THE SYSTEM IN UNFAVOURABLE CONDITIONS

We shall analyse the evolution of the system (1.2) when placed in unfavourable conditions. We shall for the time being defer the crucial problem of the relation between the quasi-biological words written below and the mathematical model and why these words describe something in biology. It is easiest to consider that this point contains a separate logical model and to leave open the question of the correspondence of the models with each other and each of them to the phenomenon to be "explained".

Let the colony of individuals be in an unfavourable medium. Two modes of adaptation are possible—one through increased stability, the other through increased adaptivity. Each of them has its own merits and drawbacks.

Increase in stability is useful to the individual in favourable conditions. However, the colony, as a whole, will occupy a smaller area since the "shock" boundary will be the death boundary for it.

Increased adaptivity reduces the activity of each individual since part of the time is spent by the adaptive individuals in the "shock" state into which they enter even in conditions favourable for more stable individuals. However, the colony of such individuals may embrace regions absolutely unsuitable for more stable individuals.

It is worth noting that the almost obvious ideas presented here are based entirely on the properties of separate individuals. The interaction of individuals plays no role in such discussion.

The next step essentially rests on interaction. We shall assume that the medium is variable (in space or time—this is immaterial—such problems are duplicates) and the individuals are capable of interacting.

In this case change in the medium results in a shift in the properties of the individuals either towards adaptivity or towards stability. The direction of this shift is determined by the type of interaction.

A number of interesting problems arise. What must the interaction be for impairment of the conditions to produce a general shift in adaptivity? Can one obtain the same results (maintenance of the viability of the colony) through passive increase in stability? Does the result depend on the rate of deterioration of the situation or simply on how far the conditions are unfavourable?

It is curious that many such problems may be analyzed in an important special case of the model when all the individuals are identical. In this case of a so-called "synchronous" or to be more exact "homogeneous" culture, the interaction may be taken care of by introducing one (or several) further variables and the problem is then that

of investigating the qualitative behaviour of sets of 3–4 equations. Such problems are quite amenable to modern analysis based (where necessary) on computer techniques.

However, of much more interest are situations in which homogeneous colonies cannot survive. Since homogeneous colonies are a particular case of heterogeneous ones then variable media are known to exist fatal for homogeneous systems but in which the existence of heterogeneous and inhomogeneous systems is possible. The simplest type of such a system is a colony some of whose individuals are shifted towards adaptivity and others towards stability. If one of the populations coincides with the whole colony then the system becomes homogeneous.

If we work close to the death boundary of homogeneous colonies then it is possible to find conditions causing splitting of the colony into an adaptive population and a stable population.

Such a situation is critical for the problem of the evolutionary origin of “control” of interest to us.

3. DIFFICULTY OF PROBLEM

It is quite clear, how the problem should be further posed. It is almost obvious that an adaptive population is a prototype of the nervous network in tissue or a population of guard ants in an anthill. The role of signal is played by the “entry” of the adaptive individuals into a shock state (or, on the contrary, emergence from it). However, even without this we have strayed too far in speculative constructions. The ideas outlined are not a theory and they cannot be verified. The situation is made even worse by the fact that conflicting facts cannot be found for it. The difficulty here is of a dual kind. Theoretical investigation of the model proposed goes beyond the bounds of modern mathematical methods. In many cases this does not scare the investigator—modern computing methods are so powerful that they may often replace theoretical analysis. Unfortunately, this is not so in our case.

Simple evaluation shows the boundless scope of the problem. For the qualitative pattern to be obtained at least in part the small parameters ε and δ must be of the order of one hundredth and not more.

$$\delta = 0.01$$

$$\varepsilon = 0.01$$

We shall select a very modest colony

$$n = 100$$

Thus, we obtain a set of 200 equations since two equations correspond to each individual. Even if we confine ourselves to paired interaction and ignore its more complex types, each of these 200 equations will contain on the right hand side 100 materially non-linear terms.

For comparison it is worth noting that even the more modest problem of integrating the set of equations of celestial mechanics—nine equations for the nine planets of the solar system—can only be done over several thousands of years.

Further calculation is impeded by the increase in the errors of rounding, “clogging” the result. We would note that 10,000 years correspond to the time

$$T = \frac{1}{\delta} = 100$$

since as a “year” in our case it is necessary to adopt the time of significant change in the fastest variable

$$\tau = \varepsilon = 0.01$$

However, in such times not only can splitting of the colony not be expected but not even any material changes in the adaptivity of the homogeneous colony.

This crude evaluation shows just how far from quantitative investigation are even the simplest evolutionary problems in an attempt to model them “head on” bypassing simplifying theoretical analysis. Similar devices with exceptionally high noise level (even the best of them do not ensure an accuracy to three symbols are absolutely unsuitable for such a problem. Therefore, it is not surprising that most investigations are confined to analysis of already existing systems with control circumventing the problem of their evolutionary origin. But even this investigation is usually based on gross simplification. For example, the approximation of the refractory medium corresponding to the cybernetic system “*control* \equiv *signal*” and taking into account only the geometry of the system.

Of course it is quite clear that in many cases such assumptions will suffice. However, the problem of the evolutionary origin of varied morphological structures, the connexion with the traversal of the critical level in evolution and the reproduction of this crisis situation in individual development is not only of fundamental but also practical importance.

The present article does not seek to give an answer to the questions posed. Its terms of reference are much more modest—to analyse the methodological roots of the abstract nature of the approaches to the problem of control in biological systems. From the point of view of mathematics the question is one of asymptotic problems with a small parameter with a major derivative, it being known from the outset that no asymptotic methods are suitable since at least a dual and even treble limiting transition is necessary. The time ($t \rightarrow \infty$) increases, the number of equations ($n \rightarrow \infty$) increases and the small parameter also tends to zero ($\varepsilon \rightarrow \infty$). At best such problems are analysed with only one limiting transition.

But if everything is so bad why write about it? The intention of the present paper is to raise the problem. Investigation of the problem in general form is at present unrealistic. However, often it happens that specific special problems can be integrated (or “analysed”) by using their special properties. A good selection of such problems sometimes replaces (in time, of course) the general theory. However, it is therefore

very important to analyse special problems of maximum diversity—the author still does not know of any—making it possible to approach the general problem from different sides.

REFERENCES

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